DC Choppers

Note Title 7/6/2015

PRINCIPLE OF STEP-DOWN OPERATION

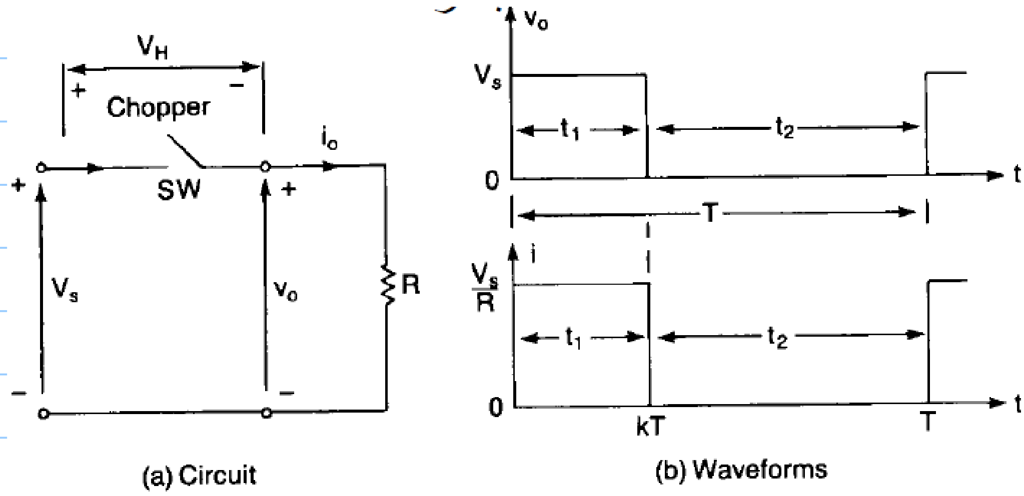


Figure 7-1 Step-down chopper with resistive load.

The average output voltage is given by

$$V_a = \frac{1}{T}\int_0^{t_1} v_0 \, dt = \frac{t_1}{T} V_s = f t_1 V_s = k V_s$$

The rms value of output voltage is found from

$$V_o = \left(\frac{1}{T}\int_0^{kT} v_0^2 \, dt\right)^{1/2} = \sqrt{k}\, V_s$$

Assuming a lossless chopper, the input power to the chopper is the same as the output power and is given by

$$P_i = \frac{1}{T}\int_0^{kT} v_0 i \, dt = \frac{1}{T}\int_0^{kT} \frac{v_0^2}{R} \, dt = k\frac{V_s^2}{R} \qquad (7\text{-}3)$$

The effective input resistance seen by the source is

$$R_i = \frac{V_s}{I_a} = \frac{V_s}{kV_s/R} = \frac{R}{k}$$

Example 7-1

The dc chopper in Fig. 7-1a has a resistive load of $R = 10\ \Omega$ and the input voltage is $V_s = 220$ V. When the chopper switch remains on, its voltage drop is $v_{ch} = 2$ V and the chopping frequency is $f = 1$ kHz. If the duty cycle is 50%, determine the (a) average output voltage, V_a; (b) rms output voltage, V_o; (c) chopper efficiency; (d) effective input resistance of the chopper, R_i; and (e) rms value of the fundamental component of output harmonic voltage.

Solution $V_s = 220$ V, $k = 0.5$, $R = 10\ \Omega$, and $v_{ch} = 2$ V.

(a) From Eq. (7-1), $V_a = 0.5 \times (220 - 2) = 109$ V.

(b) From Eq. (7-2), $V_o = \sqrt{0.5} \times (220 - 2) = 154.15$ V.

(c) The output power can be found from

$$P_o = \frac{1}{T}\int_0^{kT} \frac{v_0^2}{R}\,dt = \frac{1}{T}\int_0^{kT} \frac{(V_s - v_{ch})^2}{R}\,dt = k\,\frac{(V_s - v_{ch})^2}{R}$$

$$= 0.5 \times \frac{(220 - 2)^2}{10} = 2376.2 \text{ W}$$

The input power to the chopper can be found from

$$P_i = \frac{1}{T}\int_0^{kT} V_s i\,dt = \frac{1}{T}\int_0^{kT} \frac{V_s(V_s - v_{ch})}{R}\,dt = k\,\frac{V_s(V_s - v_{ch})}{R}$$

$$= 0.5 \times 220 \times \frac{220 - 2}{10} = 2398 \text{ W}$$

The chopper efficiency is

$$\frac{P_o}{P_i} = \frac{2376.2}{2398} = 99.09\%$$

(d) From Eq. (7-4), $R_i = 10/0.5 = 20\ \Omega$.

(e) The output voltage as shown in Fig. 7-1b can be expressed in a Fourier series as

$$v_o(t) = kV_s + \frac{V_s}{n\pi}\sum_{n=1}^{\infty} \sin 2n\pi k \cos 2n\pi ft + \frac{V_s}{n\pi}\sum_{n=1}^{\infty} (1 - \cos 2n\pi k) \sin 2n\pi ft \qquad (7\text{-}7)$$

The fundamental component (for $n = 1$) of output voltage harmonic can be determined from Eq. (7-7) as

$$v_1(t) = \frac{V_s}{\pi}[\sin 2\pi k \cos 2\pi ft + (1 - \cos 2\pi k) \sin 2\pi ft]$$
$$= \frac{220 \times 2}{\pi} \sin(2\pi \times 1000t) = 140.06 \sin(6283.2t) \qquad (7\text{-}8)$$

and its rms value is $V_1 = 140.06/\sqrt{2} = 99.04$ V.

Note. The efficiency calculation, which includes the conduction loss of the chopper, does not take into account the switching loss due to turn-on and turn-off of the chopper.

STEP-DOWN CHOPPER WITH *RL* LOAD

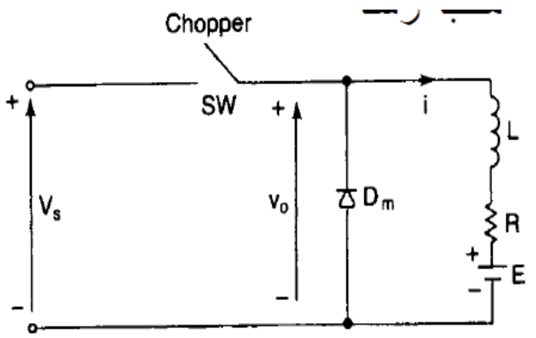


Figure 7-2 Chopper with RL loads.

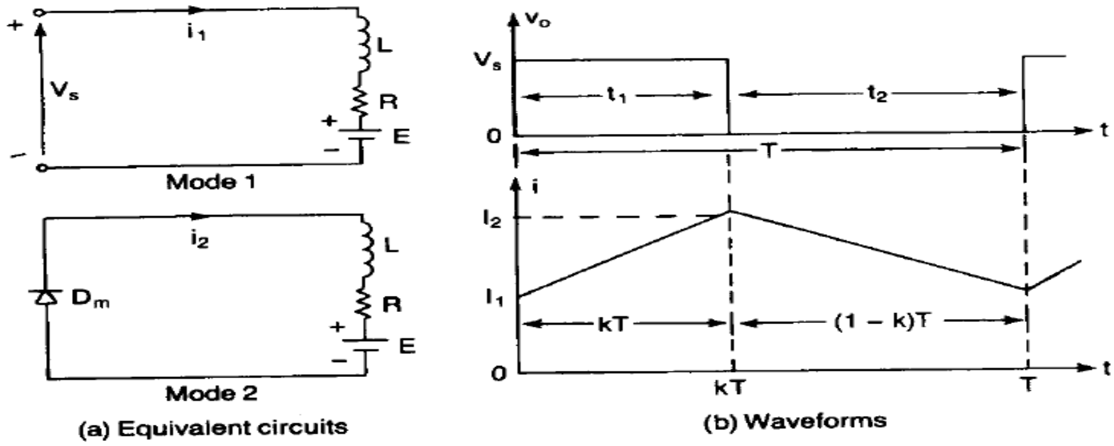


Figure 7-3 Equilvalent circuits and waveforms for RL loads.

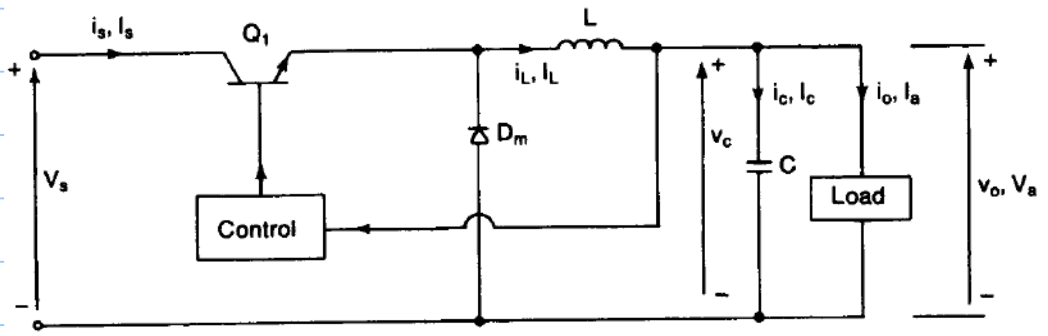
$$I_2 = I_1 e^{-kTR/L} + \frac{V_s - E}{R}(1 - e^{-kTR/L})$$

$$I_3 = I_1 = I_2 e^{-(1-k)TR/L} - \frac{E}{R}(1 - e^{-(1-k)TR/L})$$

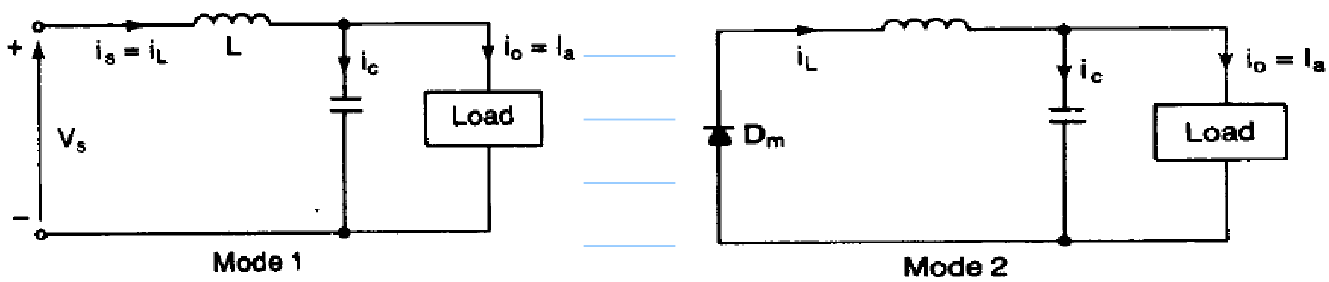
$$\Delta I_{max} = \frac{V_s}{R} \tanh \frac{R}{4fL}$$

$$\Delta I = \frac{V_s}{R} \frac{1 - e^{-kTR/L} + e^{-TR/L} - e^{-(1-k)TR/L}}{1 - e^{-TR/L}}$$

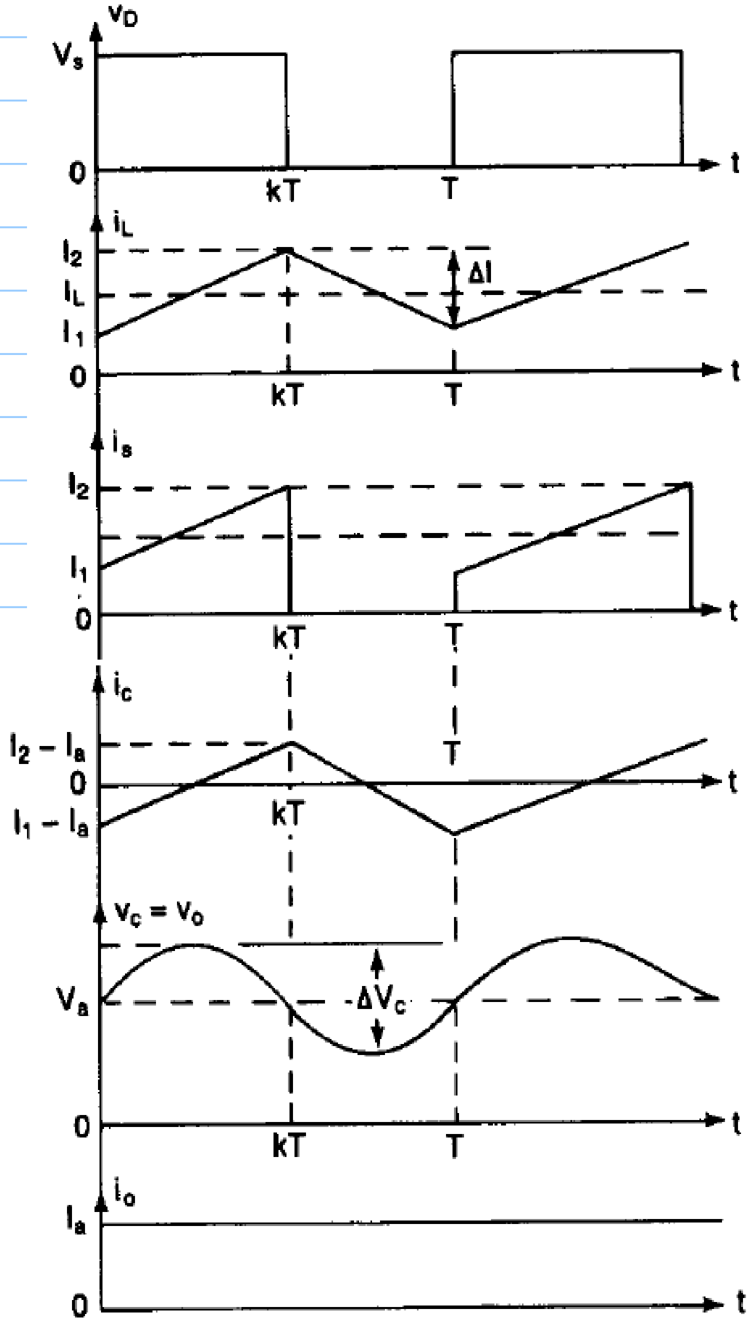
Buck Regulators



(a) Circuit diagram



(b) Equivalent circuits



(c) Waveforms

Figure 7-7 Buck regulator.

the average output voltage

$$V_a = kV_s$$

The switching period T can be expressed as

$$T = \frac{1}{f} = t_1 + t_2 = \frac{\Delta I\ L}{V_s - V_a} + \frac{\Delta I\ L}{V_a} = \frac{\Delta I\ LV_s}{V_a(V_s - V_a)}$$

which gives the peak-to-peak ripple current as

$$\Delta I = \frac{V_a(V_s - V_a)}{fLV_s}$$

or

$$\Delta I = \frac{V_s k(1 - k)}{fL}$$

The average capacitor current, which flows for $t_1/2 + t_2/2 = T/2$, is

$$I_c = \frac{\Delta I}{4}$$

the peak-to-peak ripple voltage of the capacitor is

$$\Delta V_c = v_c - v_c(t = 0) = \frac{1}{C}\int_0^{T/2} \frac{\Delta I}{4}\, dt = \frac{\Delta I\, T}{8C} = \frac{\Delta I}{8fC}$$

$$\Delta V_c = \frac{V_a(V_s - V_a)}{8LCf^2V_s}$$

or

$$\Delta V_c = \frac{V_s k(1 - k)}{8LCf^2}$$

Example 7-4

The buck regulator in Fig. 7-7a has an input voltage of $V_s = 12$ V. The required average output voltage is $V_a = 5$ V and the peak-to-peak output voltage is 20 mV. The switching frequency is 25 kHz. If the peak-to-peak ripple current of inductor is limited to 0.8 A, determine the (a) duty cycle, k; (b) filter inductance, L; and (c) filter capacitor, C.

Solution $V_s = 12$ V, $\Delta V_c = 20$ mV, $\Delta I = 0.8$ A, $f = 25$ kHz, and $V_a = 5$ V.

(a) From Eq. (7-34), $V_a = kV_s$ and $k = V_a/V_s = 5/12 = 0.4167$.

(b) From Eq. (7-37),

$$L = \frac{5(12 - 5)}{0.8 \times 25{,}000 \times 12} = 145.84\ \mu\text{H}$$

(c) From Eq. (7-39),

$$C = \frac{0.8}{8 \times 20 \times 10^{-3} \times 25{,}000} = 200\ \mu\text{F}$$

PRINCIPLE OF STEP-UP OPERATION

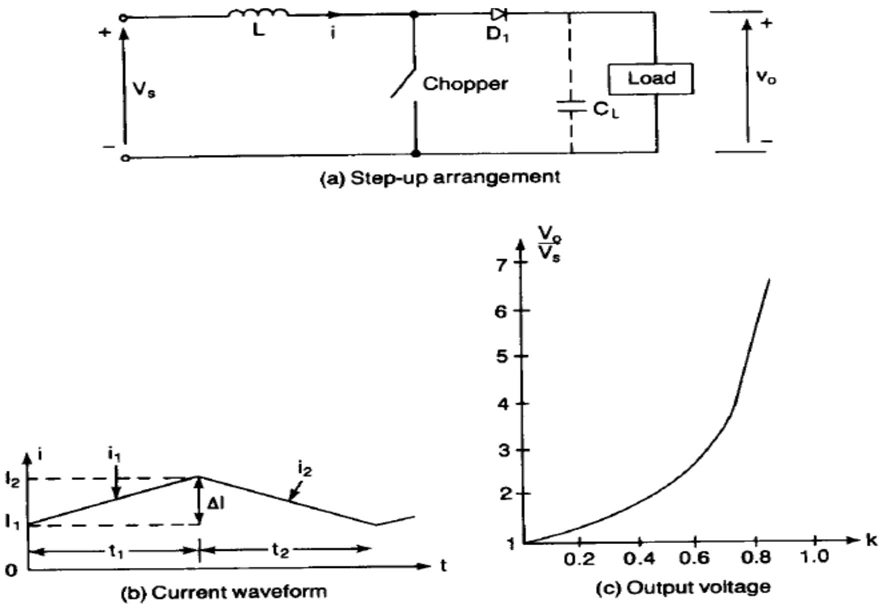


Figure 7-4 Arrangement for step-up operation.

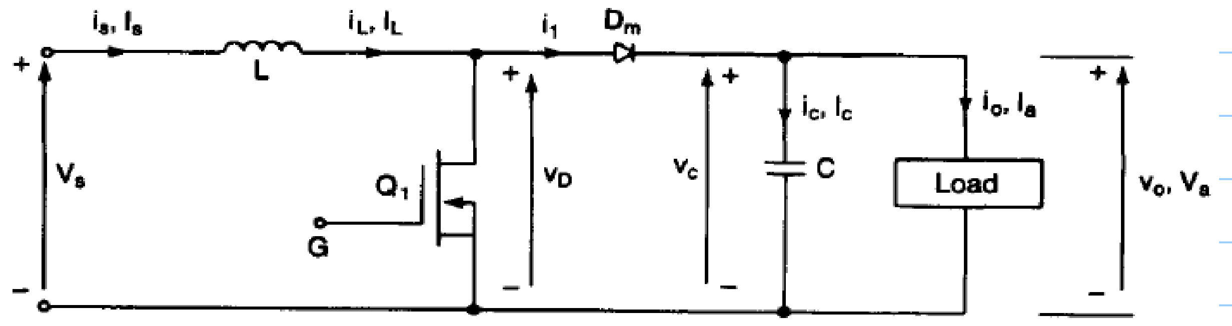
peak-to-peak ripple current in the inductor as

$$\Delta I = \frac{V_s}{L} t_1$$

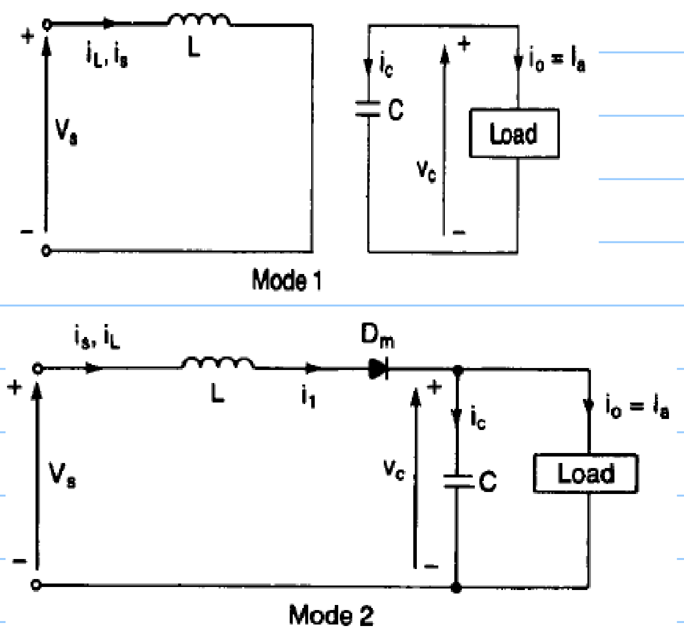
The instantaneous output voltage is

$$v_o = V_s + L\frac{\Delta I}{t_2} = V_s\left(1 + \frac{t_1}{t_2}\right) = V_s\frac{\Delta I}{1 - k}$$

Boost Regulators

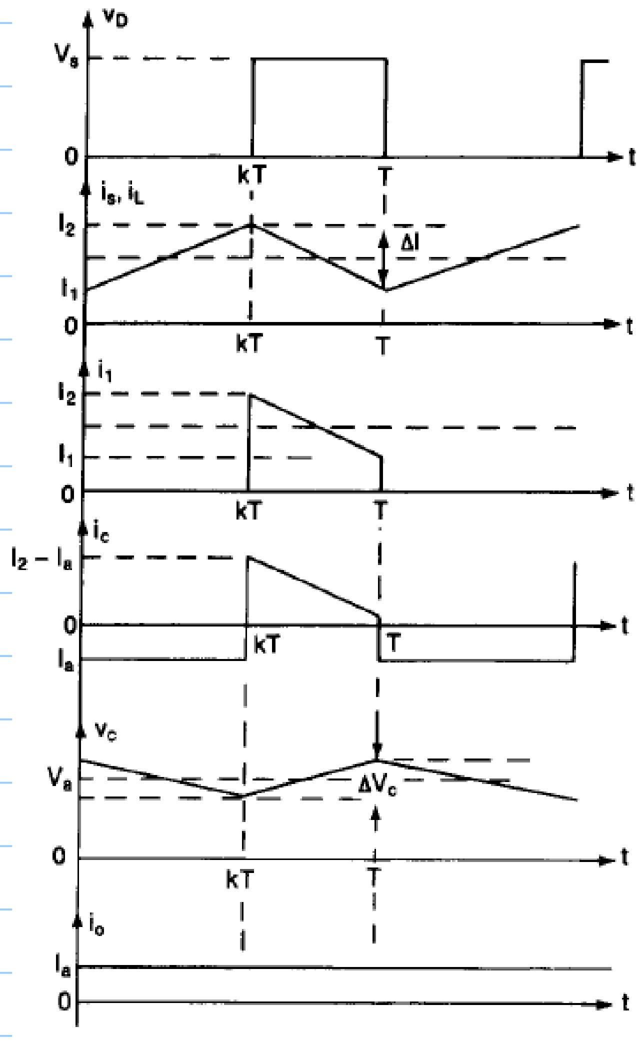


(a) Circuit diagram



(b) Equivalent circuits

$$V_s = L\frac{I_2 - I_1}{t_1} = L\frac{\Delta I}{t_1}$$



(c) Waveforms

ΔI is the peak-to-peak ripple current of inductor L

$$\Delta I = \frac{V_s t_1}{L} = \frac{(V_a - V_s)t_2}{L} = \frac{V_s(V_a - V_s)}{fLV_a} = \frac{V_s k}{fL}$$

the average input current is

$$I_s = \frac{I_a}{1 - k}$$

the average output voltage

$$V_a = \frac{V_s}{1 - k}$$

The switching period T can be found from

$$T = \frac{1}{f} = t_1 + t_2 = \frac{\Delta I\ L}{V_s} + \frac{\Delta I\ L}{V_a - V_s} = \frac{\Delta I\ LV_a}{V_s(V_a - V_s)}$$

the peak-to-peak ripple voltage of the capacitor is

$$\Delta V_c = \frac{I_a(V_a - V_s)}{V_a fC} = \frac{I_a k}{fC}$$

Example 7-5

A buck regulator in Fig. 7-8a has an input voltage of $V_s = 5$ V. The average output voltage, $V_a = 15$ V and the average load current, $I_a = 0.5$ A. The switching frequency is 25 kHz. If $L = 150$ μH and $C = 220$ μF, determine the (a) duty cycle, k; (b) ripple current of inductor, ΔI; (c) peak current of inductor, I_2; and (d) ripple voltage of filter capacitor, V_c.

Solution $V_s = 5$ V, $V_a = 15$ V, $f = 25$ kHz, $L = 150$ μH, and $C = 220$ μF.
(a) From Eq. (7-46), $15 = 5/(1 - k)$ or $k = 2/3 = 0.6667 = 66.67\%$.
(b) From Eq. (7-49),

$$\Delta L = \frac{5 \times (15 - 5)}{25{,}000 \times 150 \times 10^{-6} \times 15} = 0.89 \text{ A}$$

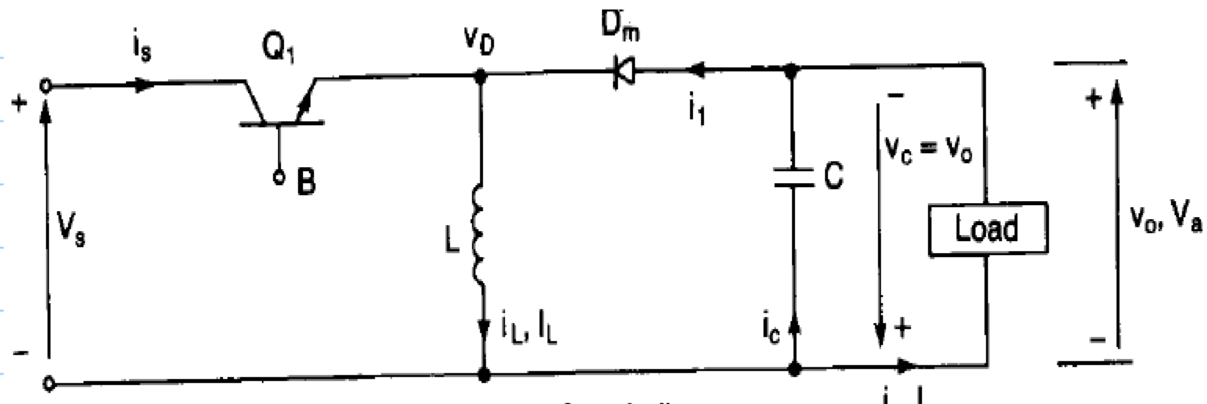
(c) From Eq. (7-47), $I_s = 0.5/(1 - 0.667) = 1.5$ A and peak inductor current,

$$I_2 = I_s + \frac{\Delta I}{2} = 1.5 + \frac{0.89}{2} = 1.945 \text{ A}$$

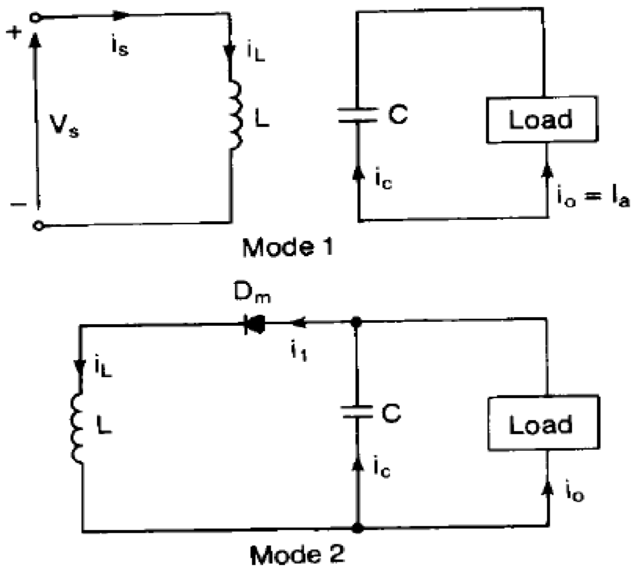
(d) From Eq. (7-53),

$$\Delta V_c = \frac{0.5 \times 0.6667}{25{,}000 \times 220 \times 10^{-6}} = 60.61 \text{ mV}$$

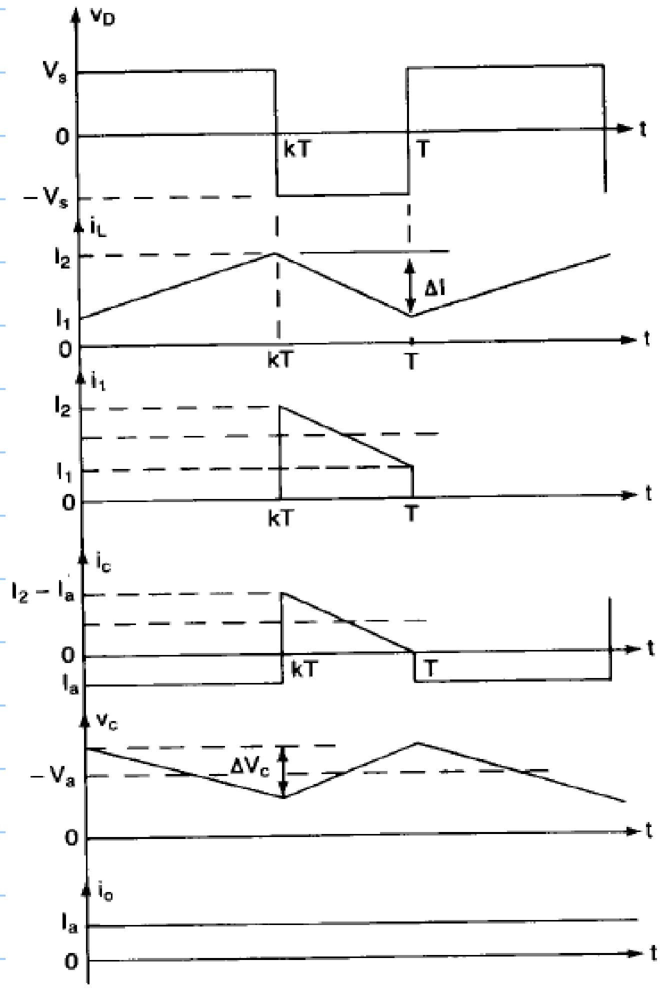
Buck–Boost Regulators



(a) Circuit diagram



(b) Equivalent circuits



(c) Waveforms

$$t_1 = \frac{\Delta I \ L}{V_s}$$

$$V_a = -L \frac{\Delta I}{t_2} = -\frac{V_s k}{1 - k}$$

$$\Delta I = \frac{V_s t_1}{L} = \frac{-V_a t_2}{L} = \frac{V_s V_a}{fL(V_a - V_s)} = \frac{V_s k}{fL}$$

$$I_s = \frac{I_a k}{1 - k}$$

$$T = \frac{1}{f} = t_1 + t_2 = \frac{\Delta I \ L}{V_s} - \frac{\Delta I \ L}{V_a} = \frac{\Delta I \ L(V_a - V_s)}{V_s V_a}$$

$$\Delta V_c = \frac{I_a V_a}{(V_a - V_s) f C} \doteq \frac{I_a k}{f C}$$

Example 7-6

The buck–boost regulator in Fig. 7-9a has an input voltage of $V_s = 12$ V. The duty cycle, $k = 0.25$ and the switching frequency is 25 kHz. The inductance, $L = 150$ μH and filter capacitance, $C = 220$ μF. The average load current, $I_a = 1.25$ A. Determine the (a) average output voltage, V_a; (b) peak-to-peak output voltage ripple, ΔV_c: (c) peak-to-peak ripple current of inductor, ΔI; and (d) peak current of the transistor, I_p.

Solution $V_s = 12$ V, $k = 0.25$, $I_a = 1.25$ A, $f = 25$ kHz, $L = 150$ μH, and $C = 220$ μF.

(a) From Eq. (7-58), $V_a = -12 \times 0.25/(1 - 0.25) = -4$V.

(b) From Eq. (7-65), the peak-to-peak output ripple voltage is

$$\Delta V_c = \frac{1.25 \times 0.25}{25{,}000 \times 220 \times 10^{-6}} = 56.8 \text{ mV}$$

(c) From Eq. (7-62), the peak-to-peak inductor ripple is

$$\Delta I = \frac{12 \times 0.25}{25{,}000 \times 150 \times 10^{-6}} = 0.8 \text{ A}$$

(d) From Eq. (7-59), $I_s = 1.25 \times 0.25/(1 - 0.25) = 0.4167$ A. Since I_s is the average of duration kT, the peak-to-peak current of the transistor,

$$I_p = \frac{I_s}{k} + \frac{\Delta I}{2} = \frac{0.4167}{0.25} + \frac{0.8}{2} = 2.067 \text{ A}$$